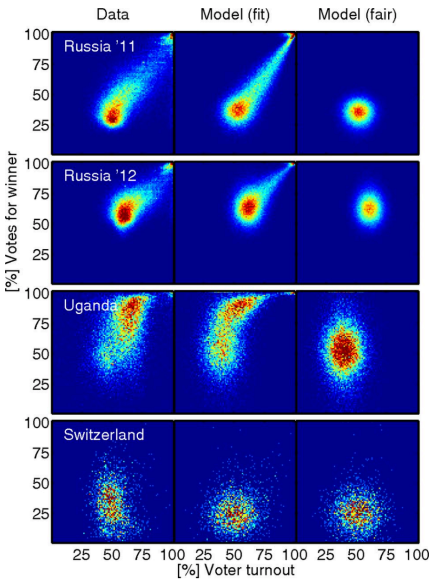
The US National Academy of Sciences (NAS) published a paper in 2012 titled “Statistical detection of systematic election irregularities.” [1] The paper asked the question, “How can it be distinguished whether an election outcome represents the will of the people or the will of the counters?” The study reviewed the results from elections in Russia and other countries, where widespread fraud was suspected. The study was published in the proceedings of the National Academy of Sciences as well as referenced in multiple election guides by USAID [2][3], among other citations.

The study authors’ thesis was that with a large sample of the voting data, they would be able to see whether or not voting patterns deviated from the voting patterns of elections where there was no fraud. The results of their study proved that there were indeed significant deviations from the expected, normal voting patterns in the elections where fraud was suspected.

Statistical results are often graphed, to provide a visual representation of how normal data should look. A particularly useful visual representation of election data is the *election fingerprint*.

When used to analyze election data, the election fingerprint typically analyzes the votes for the winner versus voter turnout by voting district. The greater the voter turnout, the more likely the result appears like a ‘fingerprint’. The expected shape of the fingerprint is of that of a 2D Gaussian (a.k.a. “Normal”) distribution.[4]

Here is an example from the referenced National Academy of Sciences paper:



The actual election results in Russia, Uganda and Switzerland appear in the left column, the right column is the expected appearance in a fair election with little fraud, and the middle column is the researchers’ model with fraud included.

As you can see, the election in Switzerland shows a range of voter turnout, from approximately 30 - 70% across voting districts, and a similar range of votes for the winner.

What do the clusters mean in the Russia 2011 and 2012 elections? Of particular concern are the top right corners, showing nearly 100% turnout of voters, and nearly 100% of them voted for the winner.

Both of those events (more than 90% of registered voters turning out to vote and more than 90% of the voters voting for the winner) are statistically improbable, even for very contested elections. Election results that show a strong linear streak away from the main fingerprint lobe indicates ‘ballot stuffing,’ where ballots are added at a specific rate. Voter turnout over 100% indicates ‘extreme fraud’. [1] [5]

Election results with ‘outliers’ - results that fall outside of normal voting patterns - are not in and of themselves definitive proof of outright fraud. But additional reviews of voting patterns and election results should be conducted whenever deviations from normal patterns occur in an election.

Using this studies methodology, in late 2020 and 2021, multiple researchers in the US have applied the same analysis to the US 2020 election results, as well as the results of previous elections.

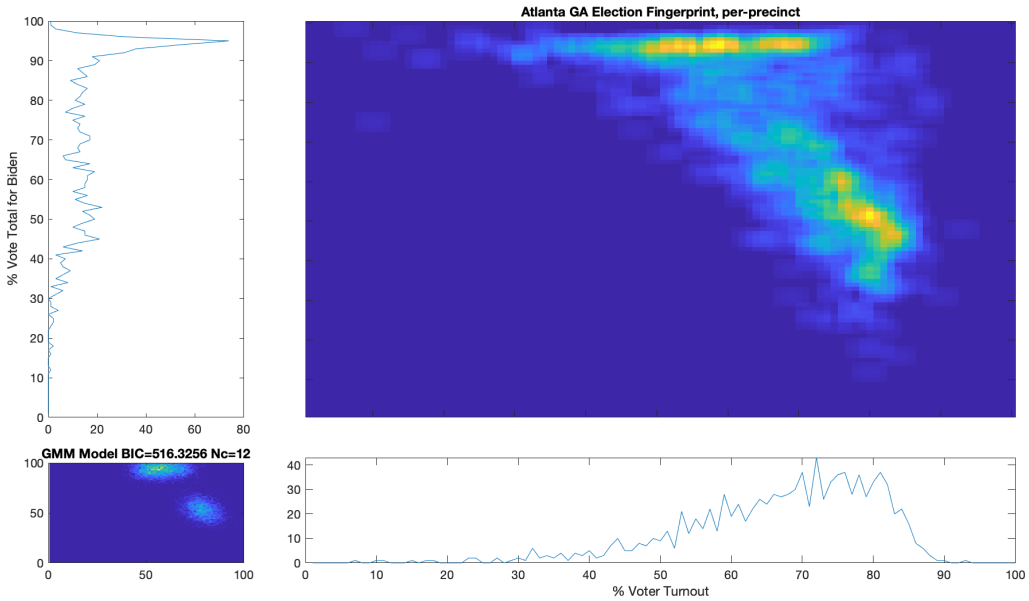
The US 2020 Election - Atlanta GA

The upper right image in the following graphic is the computed election fingerprint, computed according to the NAS paper and using official state reported voter turnout and votes for the statewide winner. The color scale moves from precincts with low counts as deep blue, to precincts with high numbers represented as bright yellow. Note that a small blurring filter was applied to the computed image for ease of viewing small isolated histogram hits.

The bottom left image of the graphic shows what an “idealized” model of the data could look like. The upper right image was computed per the NAS paper; the bottom left image shows what an idealized Gaussian model of the data should look like, based on the reported voter turnout and votes for the winner.

The top-left and bottom-right plots show the sum of the rows and columns of the fingerprint image. The top-left graph corresponds to the sum of the rows in the upper right image and is the histogram of the vote share for Biden across precincts. The bottom right plot corresponds sum of the columns of the upper right image, and is the histogram of the % turnout across the precincts.

Generated Election Fingerprint, Atlanta GA:



Conclusions:

- The results do not form a normal Gaussian distribution, and are therefore, by definition, an "irregular" distribution. The main lobe of the 'fingerprint' also has a diffuse linear streak up and to the left. According to the authors of the NAS paper, election results that show a strong streaking away from the main lobe may indicate 'ballot stuffing,' where ballots are added (or subtracted) at a specific rate. The election fingerprint is in the form of a main lobe and streak, although the streak is not as pronounced as the NAS paper's Russia results.

- Vote share near or over 100% is highly irregular and indicates a strong potential for fraud in any election. In the image above, there is a distinct and shape line across multiple turnout bins of near 100% vote share for Biden. Even in contentious elections, voter turnout over 90% is statistically unlikely, but not impossible.

- Of the 1022 polling places in Atlanta, 252 are in the > 90% vote share for Biden band, 201 had > 93%, 84 had > 95%, and 8 had > 97%.

- These findings indicate significant election irregularities that warrant additional scrutiny and investigation. It should be reiterated that the observed irregularities discussed above can serve as useful indicators and warnings of issues with an election, but do not constitute absolute proof of fraud on their own.

References:

[1] "Statistical detection of election irregularities"
Peter Klimek, Yuri Yegorov, Rudolf Hanel, Stefan Thurner Proceedings of the National Academy of Sciences Oct 2012, 109 (41) 16469-16473; DOI: 10.1073/pnas.1210722109 (https://www.pnas.org/content/109/41/16469)

[2] USAID: Assessing and Verifying Election Results: A Decision Makers Guide to Parallel Vote Tabulation and Other Tools (http://web.archive.org/web/20201118021847/https://pdf.usaid.gov/pdf_docs/PA00KGWR.pdf)

[3] USAID: A guide to Election Forensics (http://web.archive.org/web/20210501091306/https://pdf.usaid.gov/pdf_docs/PA00MXR7.pdf)

[4] Multivariate Normal Distribution - Wikipedia (https://en.wikipedia.org/wiki/Multivariate_normal_distribution)

[5] Mebane, Walter R. and Kalinin, Kirill, Comparative Election Fraud Detection (2009). APSA 2009 Toronto Meeting Paper, Available at SSRN: https://ssrn.com/abstract=1450078