

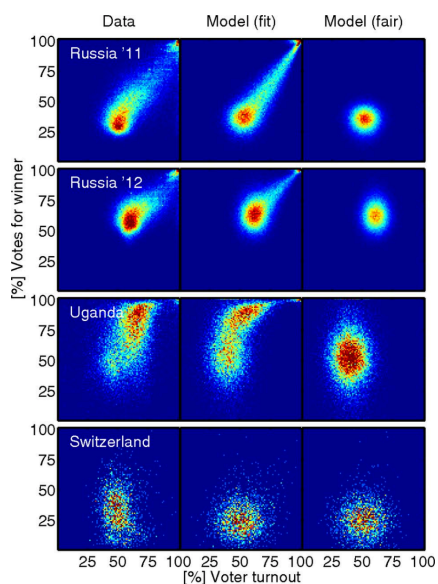
The US National Academy of Sciences (NAS) published a paper in 2012 titled “Statistical detection of systematic election irregularities.” [1] The paper asked the question, “How can it be distinguished whether an election outcome represents the will of the people or the will of the counters?” The study reviewed the results from elections in Russia and other countries, where widespread fraud was suspected. The study was published in the proceedings of the National Academy of Sciences as well as referenced in multiple election guides by USAID [2][3], among other citations.

The study authors’ thesis was that with a large sample of the voting data, they would be able to see whether or not voting patterns deviated from the voting patterns of elections where there was no fraud. The results of their study proved that there were indeed significant deviations from the expected, normal voting patterns in the elections where fraud was suspected.

Statistical results are often graphed, to provide a visual representation of how normal data should look. A particularly useful visual representation of election data is the *election fingerprint*.

When used to analyze election data, the election fingerprint typically analyzes the votes for the winner versus voter turnout by voting district. The greater the voter turnout, the more likely the result appears like a ‘fingerprint’. The expected shape of the fingerprint is of that of a 2D Gaussian (a.k.a. “Normal”) distribution.[4]

Here is an example from the referenced National Academy of Sciences paper:



The actual election results in Russia, Uganda and Switzerland appear in the left column, the right column is the expected appearance in a fair election with little fraud, and the middle column is the researchers’ model with fraud included.

As you can see, the election in Switzerland shows a range of voter turnout, from approximately 30 - 70% across voting districts, and a similar range of votes for the winner.

What do the clusters mean in the Russia 2011 and 2012 elections? Of particular concern are the top right corners, showing nearly 100% turnout of voters, and nearly 100% of them voted for the winner.

Both of those events (more than 90% of registered voters turning out to vote and more than 90% of the voters voting for the winner) are statistically improbable, even for very contested elections. Election results that show a strong linear streak away from the main fingerprint lobe indicates ‘ballot stuffing,’ where ballots are added at a specific rate. Voter turnout over 100% indicates extreme fraud. [1] [5]

Election results with 'outliers' - results that fall outside of normal voting patterns - are not in and of themselves definitive proof of outright fraud. But additional reviews of voting patterns and election results should be conducted whenever deviations from normal patterns occur in an election.

Using this studies methodology, in late 2020 and 2021, multiple researchers in the US have applied the same analysis to the US 2020 election results, as well as the results of previous elections.

### **The US 2020 Election - Wisconsin**

Election fingerprint analysis was conducted on the state of Wisconsin. Wisconsin voting is conducted by voting wards within counties.

#### **Data sources:**

1 - Wisconsin Election Day Voter Registration Data, as of November 3, 2020, <https://elections.wi.gov>

2 - Wisconsin Ward by Ward Voting Results Report, Presidential Election, November 3, 2020, <https://elections.wi.gov>

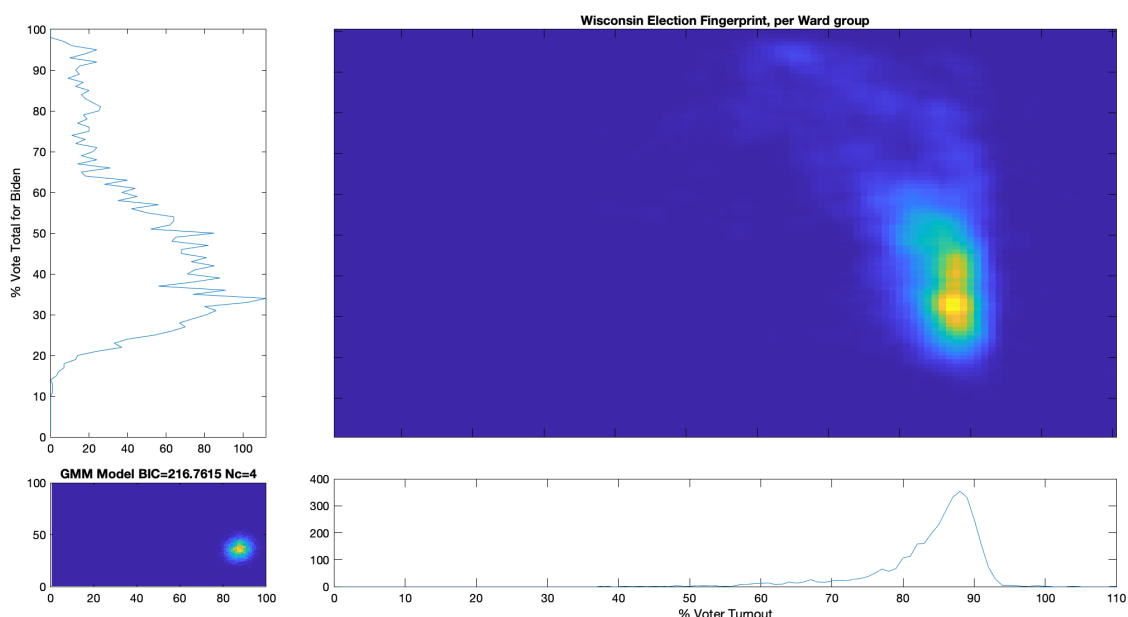
#### **Method:**

The upper right image in the following graphic is the computed statewide election fingerprint, computed according to the NAS paper and using Wisconsin's reported voter turnout and votes for the statewide winner. The color scale moves from wards with low counts as deep blue, to wards with high numbers represented as bright yellow. Note that a small blurring filter was applied to the computed image for ease of viewing small isolated histogram hits.

The bottom left image of the graphic shows what an "idealized" model of the data could look like. The upper right image was computed per the NAS paper; the bottom left image shows what an idealized Gaussian model of the data should look like, based on the reported voter turnout and votes for the winner.

The top-left and bottom-right plots show the sum of the rows and columns of the fingerprint image. The top-left graph corresponds to the sum of the rows in the upper right image and is the histogram of the vote share for Biden across wards. The bottom right plot corresponds sum of the column of the upper right image, and is the histogram of the % turnout across the wards.

### **Generated Election Fingerprint, State of Wisconsin:**



## Conclusions:

- ◆ The WI results do not form a normal Gaussian distribution, and are therefore, by definition, an “irregular” distribution. The main lobe of the ‘fingerprint’ also has a diffuse streak up and to the left.
- ◆ According to the authors of the NAS paper, election results that show a strong streaking away from the main lobe may indicate ‘ballot stuffing,’ where ballots are added (or subtracted) at a specific rate. The Wisconsin election fingerprint is in the form of a main lobe and streak, although the streak is not as pronounced as the NAS paper’s Russia results.
- ◆ The diffuse “tail” above the central lobe only begins to present itself for precincts that show Biden as the winner with over 50% vote share, with some wards reaching near 100% vote share for Biden.
- ◆ Voter turnout near or over 100% is highly irregular and indicates a strong potential for fraud in any election. In the image above, there are two faintly visible blue spots at near 100% turnout, at approximately 50% and 80% vote share for Biden.
- ◆ The upper right graph above indicates that a large proportion of wards had approximately 90% turnout in the 2020 Presidential election, and 40 - 50% of the majority of the wards voted for winner. Even in contentious elections, voter turnout over 90% is statistically unlikely, but not impossible.

- ◆ Similar analysis was conducted on a county by county basis. Of the 72 counties in Wisconsin, 23 have very significant election fingerprint deviations and irregularities, while 20 additional counties have significant election fingerprint deviations and irregularities. Seventeen specific wards had extremely high voter turnout (more than 97% of all registered voters) and/or extremely high number of votes for the winner (more than 97% of votes cast).
- ◆ These findings indicate significant election irregularities in Wisconsin that warrant additional scrutiny and investigation. It should be reiterated that the observed irregularities discussed above can serve as useful indicators and warnings of issues with an election, but do not constitute proof of fraud on their own.

**References:**

- [1] "Statistical detection of election irregularities"  
Peter Klimek, Yuri Yegorov, Rudolf Hanel, Stefan Thurner Proceedings of the National Academy of Sciences Oct 2012, 109 (41) 16469-16473; DOI: 10.1073/pnas.1210722109 (<https://www.pnas.org/content/109/41/16469>)
- [2] USAID: Assessing and Verifying Election Results: A Decision Makers Guide to Parallel Vote Tabulation and Other Tools ([http://web.archive.org/web/20201118021847/https://pdf.usaid.gov/pdf\\_docs/PA00KGWR.pdf](http://web.archive.org/web/20201118021847/https://pdf.usaid.gov/pdf_docs/PA00KGWR.pdf))
- [3] USAID: A guide to Election Forensics ([http://web.archive.org/web/20210501091306/https://pdf.usaid.gov/pdf\\_docs/PA00MXR7.pdf](http://web.archive.org/web/20210501091306/https://pdf.usaid.gov/pdf_docs/PA00MXR7.pdf))
- [4] Multivariate Normal Distribution - Wikipedia ([https://en.wikipedia.org/wiki/Multivariate\\_normal\\_distribution](https://en.wikipedia.org/wiki/Multivariate_normal_distribution))
- [5] Mebane, Walter R. and Kalinin, Kirill, Comparative Election Fraud Detection (2009). APSA 2009 Toronto Meeting Paper, Available at SSRN: <https://ssrn.com/abstract=1450078>